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Robust Fault Detection and Isolation with Unstructured Uncertainty Using Eigenstructure Assignment

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A numerical implementation is presented for the eigenstructure assignment approach to the design of a robust fault detection and isolation observer. By formulating the problem of multiple fault isolation in an eigenstructure, we develop a parametric characterization of all allowable eigenspaces, which provides a diagnostic observer capable of multiple fault isolation. In addition, a well-conditioned design of the allowable eigenvectors is shown to reduce the effect of the system unstructured uncertainty. The resulting diagnostic observer is robust to unstructured modeling errors without losing fault detection sensitivity. A procedure for the synthesis of a reduced-order diagnostic observer is also presented to increase the computational efficiency of the proposed observer.

I. Introduction

THE methodology of analytical redundancy is widely researched for economically performing fault detection and isolation (FDI) for controlled systems. In general, the FDI process consists of two main stages: residual generation and decision making. Residual signals generated on the basis of the available measurements and a mathematical model of the system should remain near zero as long as the system is operating normally; when particular faults occur, these signals deviate from zero in characteristic ways. In the decision-making stage, the faults responsible are identified by analyzing the characteristics of the residuals using logic operations or methods based on statistical decision theory. Methods based on analytical redundancy have been surveyed by Willsky,[1] Patton et al.,[2] Frank,[3,4] Isermann,[5] Gertler,[6] and Patton and Chen.[7]

Recently, mutual fault isolation, as opposed to mere fault detectability, has received much attention inasmuch as there may be several different sources of failures, especially in modern complex systems. The dedicated observer scheme (DOS) was first proposed by Clark et al.[8] to deal with the problem of isolating faults by using a bank of observers. However, the structure of the DOS seemed too large and time consuming, especially for time-critical systems. To simplify the structure of the DOS and enhance fault isolability, the concept of structured residual was introduced by Ben-Haim[9] and Gertler and Singer.[10] However, this method is not suitable for isolating multiple faults when different sources of failures occur simultaneously. An alternative approach, the fixed-direction residual, was proposed by Beard[11] to achieve multiple fault isolation. Unlike the structured residual, the fixed-direction residual confines each fault effect to a fault-specific direction. The original fixed-direction residual technique is based on frequency-domain design with filtering techniques.[11,12] Massoumnia[13] reformulated the problem of fixed-direction residuals and solved it in a geometric framework. White and Speyer[14] proceeded to improve the solution by using eigenstructure assignment. Their work showed that a powerful but simple FDI, which performs multiple fault isolation with direct numerical solutions, is required.

Approaches based on state observers are one of the most important and favored methods in designing residual generator for performing FDI.[4] However, limitations regarding accuracy, reliability, and robustness lead to several restrictions on the applicability of model-based FDI because in practice no accurate mathematical model is available. Thus, robustness has become a key issue in model-based FDI and has been discussed by Lou et al.,[15] Viswanadham and Srichander,[16] Frank,[3,4] Frank et al.,[17] and Patton and Chen,[18] all of whom considered the so-called structured uncertainties. An overview of robust issues and solutions can be found in Frank.[4] In addition to structured uncertainties, unstructured uncertainties, where both the distributions and characteristics are unknown, are inevitably present in real systems, and these uncertainties seriously corrupt the FDI procedure. A robust FDI method insensitive to unstructured modeling errors but sensitive to failures, thus, is needed.

Eigenstructure assignment has gained special attention on system performance enhancement because of its flexibility in multi-input/multi-output (MIMO) control system design.[19–21] Patton and Chen[18,22] used eigenstructure assignment on structured uncertainty decoupling by orthogonality to achieve robust fault detection, whereas we are concerned with unstructured uncertainties whose characteristics and distributions are unknown. In our design, each fault distribution is confined on one and only one specific eigenvector first; accompanied with a projection matrix a diagonal fault/residual map is then obtained, which provides a direct way to isolate multiple faults when they occur simultaneously. A well-conditioned observer eigenvector design, which Sobel and Banda[21] used to improve the robustness of the state estimate to initial condition mismatch and eigenvalue perturbation, is further extended to present work on improving the robustness of the proposed diagnostic observer to unstructured uncertainties. Results of this paper reveal that the enhancement of the FDI robustness to unstructured uncertainties will not lose any fault detection sensitivity, which has been seldom recognized before.

We note that the design freedom is defined by the number of independent outputs when employing a diagnostic observer as a residual generator. Therefore, in the derivation of this paper, the number of outputs is always assumed greater than the number of faults because the dimension of the output measurements has to provide sufficient information for isolating multiple faults and the additional freedom for characterizing the robustness of a diagnostic observer. Complete solution in this paper is provided using simple computation algorithms by which there is no longer a question of numerical complexity. Moreover, a design of the reduced-order diagnostic observer is

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also presented by retracting those states useless for fault detection, because the objective here is to diagnose faults instead of state estimation. Results from the application of this method to a vertical takeoff and landing (VTOL) aircraft are provided.

II. Fault Detection and Isolation Using Eigenstructure Assignment

A. Problem Formulation

Consider the nominal plant formulated as

$$\boldsymbol{x}(k+1) = G\boldsymbol{x}(k) + H\boldsymbol{u}(k) + K\boldsymbol{f}(k) \tag{1}$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) \tag{2}$$

where $\boldsymbol{x}(k) \in R^n$ is the state vector, $\boldsymbol{u}(k) \in R^m$ is the known control input vector, $\boldsymbol{y}(k) \in R^r$ is the vector of the measured signal, and $\boldsymbol{f}(k) \in R^p$ is the fault vector. G, H, C, and K are known matrices with proper dimensions; the term $K\boldsymbol{f}(k)$ models the fault effects acting on actuators or components. Without loss of generality, the output matrix C is assumed of full rank and the pair (C, G) is completely observable. Park and Rizzoni[23] indicated that this model also can be adopted to represent all sensor faults providing these are suitably modeled. In the following discussion, it is assumed $r > p$ to provide sufficient measurement information for isolating p multiple faults with additional freedom to characterize a diagnostic observer.

In general, a diagnostic observer for generating residuals for FDI is shown in Fig. 1. The residual generator makes use of an observer to estimate system states, to reconstruct outputs, and to generate residual signals. The observer dynamics considered can be expressed as follows:

$$\hat{\boldsymbol{x}}(k+1) = (G - LC)\hat{\boldsymbol{x}}(k) + H\boldsymbol{u}(k) + L\boldsymbol{y}(k) \tag{3}$$

$$\hat{\boldsymbol{y}}(k) = C\hat{\boldsymbol{x}}(k) \tag{4}$$

$$\boldsymbol{r}(k) = W[\boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)] \tag{5}$$

where $\boldsymbol{r}(k)$ is the residual vector for fault monitoring, $L \in R^{n\times r}$ is the observer feedback gain matrix, and W is a constant projection matrix to project residuals onto specific directions.

Defining the state estimation error $\boldsymbol{e}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$, we obtain

$$\boldsymbol{e}(k+1) = G_L\boldsymbol{e}(k) + K\boldsymbol{f}(k) \tag{6}$$

$$\boldsymbol{r}(k) = WC\boldsymbol{e}(k) \tag{7}$$

where $G_L = (G - LC)$. The complete response of the residual vector to the faults is

$$\boldsymbol{r}(z) = G_{rf}(z)\boldsymbol{f}(z) \tag{8}$$

$$G_{rf}(z) = WC(zI - G_L)^{-1}K \tag{9}$$

To detect the ith fault in the residual vector $\boldsymbol{r}(z)$, the ith column $[G_{rf}(z)]_i$ of the transfer matrix $G_{rf}(z)$ must be nonzero, especially for steady-state values, i.e.,

$$[G_{rf}(z)]_i \neq 0 \qquad \text{and} \qquad [G_{rf}(1)]_i \neq 0$$

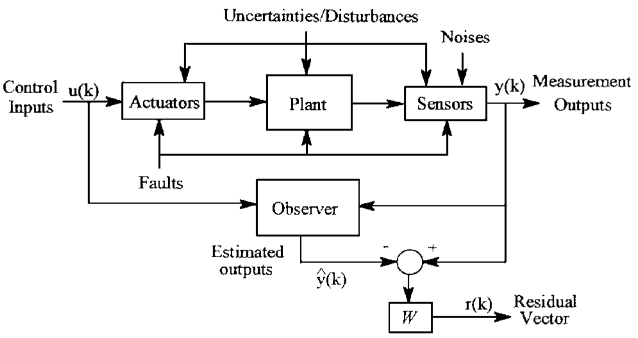

Fig. 1 Observer-based scheme for FDI.

If these conditions are satisfied, the ith fault is said to be detectable using the residuals. In particular, if $G_{rf}(z)$ is a diagonal matrix, each element of the residual is affected only by a certain fault and, thus, multiple faults can be isolated. Our work here is to design W and L so that $G_{rf}(z)$ can be in diagonal form.

B. Eigenstructure Assignment Approach for Multiple Fault Isolation

This paper first examines a numerical solution for eigenstructure assignment that yields an efficient algorithm and then parameterizes the set of all achievable eigenvectors for multiple fault isolation. Let $\boldsymbol{v}_i$ and $\boldsymbol{p}_i^T$ denote the right and left eigenvectors, respectively, associated with an eigenvalue λ_i of G_L for $i = 1, 2, \ldots, n$. We then can expand $\boldsymbol{G}_{rf}(z)$ in the eigenstructure as

$$\boldsymbol{G}_{rf}(z) = WC\left[\frac{\boldsymbol{v}_1\boldsymbol{p}_1^T}{(z-\lambda_1)} + \frac{\boldsymbol{v}_2\boldsymbol{p}_2^T}{(z-\lambda_2)} + \cdots + \frac{\boldsymbol{v}_n\boldsymbol{p}_n^T}{(z-\lambda_n)}\right]K \tag{10}$$

Equation (10) intuitively shows that multiple fault isolation can be achieved by fixed-directional residuals and, if there are $(n - \boldsymbol{p})$ left eigenvectors of $(TG - LC)$, can be assigned to be orthogonal to the columns of fault matrix K and that the entries of $\boldsymbol{p}$ faults are one-on-one projected onto the reset $\boldsymbol{p}$ left eigenvectors along the direction of $WC\bar{\boldsymbol{v}}_i$ independently. The following theorem considers how to choose the left eigenvectors $\boldsymbol{p}_i^T$ of $(G - LC)$ by eigenstructure assignment to fix the direction of the residual vector.

Theorem 1: Let k_i be the ith column of matrix K and w_i^T be the ith row of W. If the observer feedback gain L in Eq. (3) is chosen to satisfy, first,

$$\boldsymbol{p}_i^T\boldsymbol{k}_i \begin{cases} = 0, & \text{for} \quad i \neq j, \\ \neq 0, & \text{for} \quad i = j, \end{cases} \qquad \forall i, j = 1, 2, \ldots, p \tag{11}$$

and, second,

$$\boldsymbol{p}_i^T K = 0, \qquad \forall i = p+1, \ldots, n \tag{12}$$

then Eq. (10) can be obtained as

$$\boldsymbol{G}_{rf}(z) = W\left[C\boldsymbol{v}_1\frac{\langle\boldsymbol{p}_1, \boldsymbol{k}_1\rangle}{(z-\lambda_1)} \quad C\boldsymbol{v}_2\frac{\langle\boldsymbol{p}_2, \boldsymbol{k}_2\rangle}{(z-\lambda_2)} \quad \cdots \quad C\boldsymbol{v}_p\frac{\langle\boldsymbol{p}_p, \boldsymbol{k}_p\rangle}{(z-\lambda_p)}\right] \tag{13}$$

where $\langle\cdot,\cdot\rangle$ is defined as the inner product, i.e., $\langle\boldsymbol{p}_i, \boldsymbol{k}_i\rangle = \boldsymbol{p}_i^T\boldsymbol{k}_i$.

Proof: Because $\boldsymbol{p}_i^T K = 0$, $\forall i = p+1, \ldots, n$, then Eq. (10) becomes

$$\boldsymbol{G}_{rf}(z) = W\left[\sum_{i=1}^{p}\frac{C\boldsymbol{v}_i\boldsymbol{p}_i^T\boldsymbol{k}_1}{(z-\lambda_i)} \quad \sum_{i=1}^{p}\frac{C\boldsymbol{v}_i\boldsymbol{p}_i^T\boldsymbol{k}_2}{(z-\lambda_i)} \quad \cdots \quad \sum_{i=1}^{p}\frac{C\boldsymbol{v}_i\boldsymbol{p}_i^T\boldsymbol{k}_p}{(z-\lambda_i)}\right] \tag{14}$$

Moreover,

$$\boldsymbol{p}_i^T\boldsymbol{k}_i \begin{cases} = 0, & \text{for} \quad i \neq j, \\ \neq 0, & \text{for} \quad i = j, \end{cases} \qquad \forall i, j = 1, 2, \ldots, p$$

entails that Eq. (13) holds. □

It can be seen that if the $C\boldsymbol{v}_i$ for $i = 1, 2, \ldots, \boldsymbol{p}$ are linearly independent, Eq. (13) is then the so-called fixed-direction residual, in which each fault is confined to a different direction. Let $V_1 = [\boldsymbol{v}_1 \ \ \boldsymbol{v}_2 \ \ \cdots \ \ \boldsymbol{v}_p]$; it can be easily shown that CV_1 has full column rank iff $N(C) \cap R(V_1) = \{0\}$, where $N(\,)$ denotes the null space and $R(\,)$ denotes the range space. We now consider how to design the projection matrix W to diagonalize the fault/residual map.

Lemma 1: Let $P_2 = [\boldsymbol{p}_{p+1} \ \ \boldsymbol{p}_{p+2} \ \ \cdots \ \ \boldsymbol{p}_n]$. If $N(C) \cap N(P_2^T) = \{0\}$, then there exists a constant matrix $W \in R^{p\times r}$ satisfying

$$w_i^T C\boldsymbol{v}_j = \begin{cases} 0, & \text{for} \quad i \neq j, \\ \omega_i, & \text{for} \quad i = j, \end{cases} \qquad \forall i, j = 1, 2, \ldots, p \tag{15}$$

where ω_i is an arbitrary nonzero constant, and it yields

$$G_{rf}(z) = \begin{bmatrix} \dfrac{\omega_1 p_1^T k_1}{(z-\lambda_1)} & 0 & \cdots & 0 \\ 0 & \dfrac{\omega_2 p_2^T k_2}{(z-\lambda_2)} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \dfrac{\omega_p p_p^T k_p}{(z-\lambda_p)} \end{bmatrix} \tag{16}$$

Proof: Let $P_1 = [p_1 \ p_2 \ \cdots \ p_p]$ and $V_2 = [v_{p+1} \ v_{p+2} \ \cdots \ v_n]$, $P = [P_1 \ P_2]$, and $V = [V_1 \ V_2]$. Because $P^T V = I$, it implies that $R(V_1) = N(P_2^T)$. Moreover, if $N(C) \cap N(P_2^T) = \{0\}$, we have $N(C) \cap R(V_1) = \{0\}$, which shows that CV_1 has full column rank. Let $Y = CV_1 \in R^{r\times p}$. Because CV_1 has full column rank, there exists a $\bar{Y} \in R^{r\times(r-p)}$ such that $[Y \ \bar{Y}]$ is invertable. By choosing

$$W = \begin{bmatrix} \Lambda_W & 0_{p\times(r-p)} \end{bmatrix} [Y \quad \bar{Y}]^{-1} \tag{17}$$

where $W \in R^{p\times r}$ and $\Lambda_W = \mathrm{diag}(\omega_1, \omega_2, \ldots, \omega_p)$ with $\forall \omega_i \neq 0$, we obtain

$$Y = WCV_1 = \mathrm{diag}(\omega_1, \omega_2, \ldots, \omega_p) \tag{18}$$

which implies that Eq. (15) is true and, consequently, Eqs. (13) and (18) give Eq. (16). □

Note that a direct design of W can be given as

$$W = \Lambda_W (CV_1)^+ \tag{19}$$

where $(CV_1)^+ = [(CV_1)^T (CV_1)]^{-1} (CV_1)^T$ is the Moore–Penrose generalized inverse of CV_1. Obviously, by diagonalizing the $G_{rf}(z)$ in the form shown in Eq. (16), multiple fault isolation is achieved.

Remark 1: By Eq. (16), if we have

$$\omega_i = \frac{(1-\lambda_i)}{\langle p_i, k_i \rangle} \tag{20}$$

the transfer matrix from the faults to the residual vector $G_{rf}(z)$ is diagonalized with unit steady-state gain.

Remark 2: Although the preceding derivations are based on the assumption that the pair (C, G) is completely observable, the results are still valid even if the system is not completely observable. Suppose that the pair (C, G) has $\bar{o}$ unobservable modes. We have $Cv_i = 0$ when (λ_i, v_i) is an unobservable eigenpair.[24] Partition the eigenvalues of G_L into two subsets: the observable subset $\{\lambda_1, \ldots, \lambda_{n-\bar{o}}\}$ and the unobservable subset $\{\lambda_{n-\bar{o}+1}, \ldots, \lambda_n\}$. Because $Cv_i = 0$ provides an equivalent decoupling effect from the faults onto the ith mode for $i = n - \bar{o} + 1, \ldots, n$, the constraint in Eq. (12) can be relaxed as follows:

$$p_i^T K = 0, \qquad \forall i = p+1, \ldots, n - \bar{o} \tag{21}$$

which means that all of the preceding derivations are still valid.

C. Computation of Allowable Subspaces of Eigenvectors: Numerical Solution

To isolate multiple faults, in Theorem 1 we have described how to assign the left eigenvectors to fix the direction of the residual vector and in Lemma 1 we showed how to choose W to diagonalize the residual/fault map. We now consider how to construct eigenvector subspaces fulfilling Eqs. (11) and (12) by numerical treatment and obtain the numerical solution of L. Moore[19] showed that the choice of the eigenvalue λ_i determines the allowable subspace in which any chosen eigenvector must reside. Also, once the eigenvectors are determined, then the observer feedback gain matrix L can be obtained by using the following proposition.[19]

Proposition 1: Given the system representation $R = [G, H, C, 0]$ and $L \in R^{n\times r}$, suppose that $\mathcal{L} = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ denotes a distinct self-conjugate eigenvalue set of $(G - LC)$ including all unobservable eigenvalues of the pair (C, G). Let $S_{\lambda_i} = [\lambda_i I - G^T \ \ C^T]$ and let there be a compatibly partitioned matrix

$$\Sigma_{\lambda_i} = \begin{bmatrix} N_{\lambda_i} \\ M_{\lambda_i} \end{bmatrix} \tag{22}$$

whose columns constitute the basis of $N(S_{\lambda_i})$. Then, $\exists \alpha_i \neq 0$ and $p_i = N_{\lambda_i} \alpha_i$ for $i = 1, 2, \ldots, n$ such that $\{p_1 \ p_2 \ \cdots \ p_n\}$ are linearly independent and $p_i^T[\lambda_i I - (G - LC)] = 0$ with $L = [p_1 \ \cdots \ p_n]^{-T} [\xi_1 \ \cdots \ \xi_n]^T$, where $\xi_i = M_{\lambda_i} \alpha_i$.

The numerical procedure that fulfills Eqs. (11) and (12) will be simplified by applying a coordinate transformation as follows.

Theorem 2: Suppose that K has full column rank. Let $\bar{K} \in R^{n\times(n-p)}$ such that $[K \ \bar{K}]$ is invertable and $T_K = [K \ \bar{K}]^{-1}$. If

$$\tilde{p}_i = \begin{bmatrix} e_i^p \\ \tilde{\tilde{p}}_i \end{bmatrix} \quad \text{for} \quad i = 1, 2, \ldots, p \tag{23}$$

$$\tilde{p}_i = \begin{bmatrix} 0_{p\times 1} \\ \tilde{\tilde{p}}_i \end{bmatrix} \quad \text{for} \quad i = p+1, \ p+2, \ldots, n \tag{24}$$

$$p_i = T_K^T \tilde{p}_i \quad \text{for} \quad i = 1, 2, \ldots, n \tag{25}$$

where e_i^p denotes the ith column of a $p \times p$ dimension identity matrix and $\tilde{\tilde{p}}_i \in R^{(n-p)}$ is arbitrary. Then

$$p_i^T k_j = \begin{cases} 1, & \text{if} \quad i = j, \\ 0, & \text{if} \quad i \neq j, \end{cases} \quad \text{for} \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, p$$

Proof: Because $T_K = [K \ \bar{K}]^{-1}$, we have

$$T_K K = \begin{bmatrix} I_p \\ 0 \end{bmatrix} \equiv \begin{bmatrix} e_1^n & e_2^n & \cdots & e_p^n \end{bmatrix} \tag{26}$$

where e_i^n denotes the ith column of an $n \times n$ dimension identity matrix. Thus, we have

$$\begin{aligned} p_i^T k_j &= \tilde{p}_i^T (T_K k_j) \\ &= \begin{cases} 1, & \text{if} \quad i = j, \\ 0, & \text{if} \quad i \neq j, \end{cases} \quad \forall i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, p \end{aligned}$$

which completes the proof. □

Theorem 3: Suppose that there is a nonsingular linear transformation matrix T_K. Let $\tilde{G} = T_K G T_K^{-1}$ and $\tilde{C} = C T_K^{-1}$, and let the triple $(\lambda_i, \tilde{p}_i, \tilde{\xi}_i)$ satisfy

$$\begin{bmatrix} \lambda_i I - \tilde{G}^T & \tilde{C}^T \end{bmatrix} \begin{bmatrix} \tilde{p}_i \\ \tilde{\xi}_i \end{bmatrix} = 0, \qquad \text{for} \quad i = 1, 2, \ldots, n \tag{27}$$

Then

$$\begin{bmatrix} \lambda_i I - G^T & C^T \end{bmatrix} \begin{bmatrix} p_i \\ \xi_i \end{bmatrix} = 0 \tag{28}$$

if $p_i = T_K^T \tilde{p}_i$ and $\xi_i = \tilde{\xi}_i$.

Proof: Equation (27) can be rewritten as

$$\left(\lambda_i I - \tilde{G}^T\right) \tilde{p}_i + \tilde{C}^T \tilde{\xi}_i = 0 \tag{29}$$

$$\left(\lambda_i T_K^{-T} T_K^T - T_{\bar{K}}^{-T} G^T T_K^T\right) \tilde{p}_i + T_K^{-T} C^T \tilde{\xi}_i = 0 \tag{30}$$

$$T_K^{-T} \left\{ \left(\lambda_i T_K^T \tilde{p}_i - G^T T_K^T \tilde{p}_i\right) + C^T \tilde{\xi}_i \right\} = 0 \tag{31}$$

Because T_K is nonsingular, then

$$\left(\lambda_i T_K^T \tilde{p}_i - G^T T_K^T \tilde{p}_i\right) + C^T \tilde{\xi}_i = 0 \tag{32}$$

Given $p_i = T_K^T \tilde{p}_i$ and $\xi_i = \tilde{\xi}_i$, Eq. (32) can be rewritten as

$$\left(\lambda_i I - G^T\right) p_i + C^T \xi_i = 0 \tag{33}$$

which implies that Eq. (28) holds. □

By applying the linear transformation $T_K = [K \quad \bar{K}]^{-1}$, Theorems 2 and 3 provide a clear formulation of the allowable subspaces of eigenvectors so that the FDI requirements in Eq. (16) can be satisfied. Let the compatibly partitioned matrix

$$\tilde{\Sigma}_{\lambda_i} = \begin{bmatrix} \tilde{N}_{\lambda_i} \\ \tilde{M}_{\lambda_{ii}} \end{bmatrix} \tag{34}$$

whose columns constitute the bases of $N([\lambda_i I - \tilde{G}^T \quad \tilde{C}^T])$. Proposition 1 shows that

$$\tilde{\boldsymbol{p}}_i = \tilde{N}_{\lambda_i}\boldsymbol{\alpha}_i, \qquad \tilde{\xi}_i = \tilde{M}_{\lambda_i}\boldsymbol{\alpha}_i \tag{35}$$

where $\boldsymbol{\alpha}_i$ is an arbitrary nonzero column vector. As a result, the allowable eigenvectors and the observer feedback gain matrix L of the proposed FDI can be parameterized by $\boldsymbol{\alpha}_i$. To arrive at the parameterization, we first use T_K to transform the original plant (G, H, K, C) into $(\tilde{G}, \tilde{H}, \tilde{K}, \tilde{C})$ and partition $\tilde{G}$ and $\tilde{C}$ as

$$\tilde{G} = \begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{12} \\ \tilde{G}_{21} & \tilde{G}_{22} \end{bmatrix} \quad \text{and} \quad \tilde{C} = [\tilde{C}_1 \quad \tilde{C}_2] \tag{36}$$

where $\tilde{G}_{11} \in R^{p\times p}$, $\tilde{G}_{12} \in R^{p\times(n-p)}$, $\tilde{G}_{21} \in R^{(n-p)\times p}$, and $\tilde{G}_{22} \in R^{(n-p)\times(n-p)}$; and $\tilde{C}_1 \in R^{r\times p}$ and $\tilde{C}_2 \in R^{r\times(n-p)}$.

1. Decoupling Undesired Direction from Fault Effects

In the case where $i = p+1,\ldots,n$, Eq. (24) entails that Eq. (27) can be rewritten as

$$\begin{bmatrix} -\tilde{G}_{21}^T & \tilde{C}_1^T \\ \lambda_i I_{n-p} - \tilde{G}_{22}^T & \tilde{C}_2^T \end{bmatrix} \begin{bmatrix} \tilde{\tilde{\boldsymbol{p}}}_i \\ \tilde{\boldsymbol{\xi}}_i \end{bmatrix} = 0 \tag{37}$$

Define

$$\hat{S}_{\lambda_i} \equiv \begin{bmatrix} -\tilde{G}_{21}^T & \tilde{C}_1^T \\ \lambda_i I_{n-p} - \tilde{G}_{22}^T & \tilde{C}_2^T \end{bmatrix} \tag{38}$$

and the compatibly partitioned matrix

$$\hat{\Sigma}_{\lambda_i} = \begin{bmatrix} \hat{N}_{\lambda_i} \\ \hat{M}_{\lambda_i} \end{bmatrix} \tag{39}$$

whose columns constitute the bases of $N(\hat{S}_{\lambda_i})$. Then $\tilde{M}_{\lambda_i} = \hat{M}_{\lambda_i}$ and $\tilde{N}_{\lambda_i}$ is given as

$$\tilde{N}_{\lambda_i} = \begin{bmatrix} 0 \\ \hat{N}_{\lambda_i} \end{bmatrix} \tag{40}$$

□

2. Confining Fault Effects to Fixed Directions

In the case where $i = 1, 2, \ldots, \boldsymbol{p}$, let $\tilde{g}_i^T$ denote the ith row of $\tilde{G}$. Then, by Eq. (23), we can rewrite Eq. (27) in the following formulation:

$$\begin{bmatrix} \lambda_i e_i^n - \tilde{g}_i & \hat{S}_{\lambda_i} \end{bmatrix} \begin{bmatrix} 1 \\ \boldsymbol{b}_i \end{bmatrix} = 0 \tag{41}$$

where

$$\boldsymbol{b}_i = \begin{bmatrix} \tilde{\tilde{\boldsymbol{p}}}_i \\ \tilde{\boldsymbol{\xi}}_i \end{bmatrix}$$

Let $\hat{S}_{\lambda_i}^i = [\lambda_i e_i^n - \tilde{g}_i \quad \hat{S}_{\lambda_i}]$ and the compatibly partitioned matrix

$$\hat{\Sigma}_{\lambda_i}^i = \begin{bmatrix} \hat{N}_{\lambda_i}^i \\ \hat{M}_{\lambda_i}^i \end{bmatrix} \tag{42}$$

whose columns constitute the bases of $N(\hat{S}_{\lambda_i}^i)$. Then $\tilde{N}_{\lambda_i}$ can be reconstructed as

$$\tilde{N}_{\lambda_i} = \begin{bmatrix} \Delta_i^p \hat{N}_{\lambda_i}^i \\ {}_1\hat{N}_{\lambda_i}^i \end{bmatrix}, \qquad i = 1, 2, \ldots, \boldsymbol{p} \tag{43}$$

where $\Delta_i^p = [\boldsymbol{e}_i^p \quad 0_{p\times(n-p)}]$ and ${}_1\hat{N}_{\lambda_i}^i$ is a matrix constituting the rows of $\hat{N}_{\lambda_i}^i$, except the first row. □

Theorem 2 shows that the subspaces of eigenvectors fulfilling Eqs. (11) and (12) with respect to eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ can be obtained with $N_{\lambda_i} = T_K^T \tilde{N}_{\lambda_i}$ and $M_{\lambda_i} = \tilde{M}_{\lambda_i}$. From Proposition 1, which indicates that $\boldsymbol{p}_i = M_{\lambda_i}\boldsymbol{\alpha}_i$ and $\xi_i = N_{\lambda_i}\boldsymbol{\alpha}_i$, we can parameterize the set of all allowable eigenvectors by $\boldsymbol{\alpha}_i$. The observer feedback gain matrix, thus, can be computed directly from Proposition 1.

D. Numerical Algorithm of Allowable Eigenspaces

The following algorithm summarizes the preceding derivations with two major subprocedures: steps 1–3 constitute a procedure to obtain the allowable subspaces of eigenstructure assignment for multiple fault isolation and steps 5 and 6 provide a calculation of gain matrices L and W. Step 4 is one of the ways of determining a eigenstructure from the solution space.

Algorithm 1:

1) Determine a distinct self-conjugate eigenvalue set of $(G - LC)$.

2) Given $T_K = [K \quad \bar{K}]^{-1}$, where $\bar{K} \in R^{n\times(n-p)}$ is an orthonormal basis of $N(K^T)$, compute $\tilde{G} = T_K G T_K^{-1}$ and $\tilde{C} = C T_K^{-1}$.

3) For $i = p+1, \ldots, n$, a) find the basis

$$\hat{\Sigma}_{\lambda_i} = \begin{bmatrix} \hat{N}_{\lambda_i} \\ \hat{M}_{\lambda_i} \end{bmatrix}$$

of $N(\hat{S}_{\lambda_i})$, where $\hat{S}_{\lambda_i}$ is defined as in Eq. (38) and b) set

$$\tilde{N}_{\lambda_i} = \begin{bmatrix} 0 \\ \hat{N}_{\lambda_i}^i \end{bmatrix}$$

For $i = 1, 2, \ldots, p$, a) compute $\hat{S}_{\lambda_i}^i = [\lambda_i e_i - \tilde{g}_i \quad \hat{S}_{\lambda_i}]$ and find the basis

$$\hat{\Sigma}_{\lambda_i}^i = \begin{bmatrix} \hat{N}_{\lambda_i}^i \\ \hat{M}_{\lambda_i}^i \end{bmatrix}$$

of $N(\hat{S}_{\lambda_i}^i)$ and b) let $\tilde{N}_{\lambda_i}$ as

$$\tilde{N}_{\lambda_i} = \begin{bmatrix} \Delta_i^p \hat{N}_{\lambda_i}^i \\ {}_1\hat{N}_{\lambda_i}^i \end{bmatrix}$$

Obtain the assignable subspace by $M_{\lambda_i} = \hat{M}_{\lambda_i}$ and $N_{\lambda_i} = T_K^T \tilde{N}_{\lambda_i}$ for $i = 1, 2, \ldots, n$.

4) Arbitrarily determine nonzero column vector $\boldsymbol{\alpha}_i$ and calculate $\boldsymbol{p}_i = N_{\lambda_i}\boldsymbol{\alpha}_i$ and $\xi_i = M_{\lambda_i}\boldsymbol{\alpha}_i$ for $i = 1, 2, \ldots, n$ such that a) $P = [\boldsymbol{p}_1 \ \boldsymbol{p}_2 \ \cdots \ \boldsymbol{p}_n]$ linearly independent, b) $N(C) \cap N([\boldsymbol{p}_{p+1} \ \boldsymbol{p}_{p+2} \ \cdots \ \boldsymbol{p}_n]^T) = \{0\}$, and c) $\Xi = [\boldsymbol{\xi}_1 \ \cdots \ \boldsymbol{\xi}_n]$.

5) Compute $L = P^{-T}\Xi^T$.

6) Given $\Lambda_W = \mathrm{diag}(\omega_1, \omega_2, \ldots, \omega_p)$ where $\omega_i = (1 - \lambda_i)/\langle \boldsymbol{p}_i, \boldsymbol{k}_i \rangle$, calculate the required weighting matrix

$$W = \Lambda_W (C V_1)^+ = \Lambda_W \left[(C V_1)^T (C V_1)\right]^{-1} (C V_1)^T$$

After L and W are designed, a diagnostic observer is obtained as expressed in Eqs. (3–5).

The resulting parameterization in step 4 provides degrees of freedom to further enhance the robustness of the diagnostic observer to unstructured uncertainty if proper $\boldsymbol{\alpha}_i$ can be chosen.

III. Well-Conditioned Design for Robust FDI

Like many other residual generation methods, the described method employs an ideal model of the dynamic process. In most practical systems, however, uncertainties are almost inevitably present and may seriously interfere with the FDI procedure. Thus, the FDI system has to be made insensitive to modeling errors without losing fault detection sensitivity. One of the most powerful ways to achieve robustness in FDI is to make use of disturbance decoupling.[4,7] In those approaches, all uncertainties acting on a system model are modeled as unknown inputs with a known distribution matrix (the so-called structured uncertainties), on which the FDI system is designed. However, in most practical situations the structure of the uncertainty is unknown, i.e., the uncertainty is unstructured. In such cases, the robustness problem is more difficult to solve. We note that the result in preceding section can also be used to the design of unknown input decoupling diagnostic observer with a slight modification, which we do not address in this paper.

Sobel and Banda[21] presented a method of reducing the condition number of observer eigenvectors to improve the state estimation error due to initial condition mismatch and the numerical perturbation on eigenvalues. This idea is extended here to our case of diagnostic robustness to unstructured uncertainties. Consider the following disturbed fault-free system:

$$\boldsymbol{x}(k+1) = [G + \Delta G(\boldsymbol{x},\boldsymbol{u})]\cdot\boldsymbol{x}(k) + [H + \Delta H(\boldsymbol{x},\boldsymbol{u})]\cdot[\boldsymbol{u}(k)+\boldsymbol{v}(k)] \tag{44}$$

$$\boldsymbol{y}(k) = C\boldsymbol{x}(k) + w(k) \tag{45}$$

where $\boldsymbol{v}(k)$ and $w(k)$ represent the input and output disturbances and noises, respectively, and $\Delta G(\boldsymbol{x},\boldsymbol{u})$ and $\Delta H(\boldsymbol{x},\boldsymbol{u})$ formulate the unstructured uncertainty, including modeling errors, uncertainties, and nonlinearity, whose characteristics are unknown. The error and residual dynamics of the fault diagnostic observer in Eq. (6) and (7) then become

$$\boldsymbol{e}(k+1) = (G - LC)\boldsymbol{e}(k) + \Delta\boldsymbol{e}(k) \tag{46}$$

$$\boldsymbol{r}(k) = WC\boldsymbol{e}(k) + Ww(k) \tag{47}$$

where $\Delta\boldsymbol{e}(k) = \Delta G(\boldsymbol{x},\boldsymbol{u})\boldsymbol{x}(k) + \Delta H(\boldsymbol{x},\boldsymbol{u})[\boldsymbol{u}(k)+\boldsymbol{v}(k)] - Lw(k)$. One can see that the diagnostic observer of Eqs. (3–5) is no longer reliable for fault detection when uncertainties exist. To simultaneously achieve robustness to unstructured uncertainties and multiple fault isolation, the observer feedback gain L must be appropriately specified. Equations (46–47) lead to the following time-response equation for the fault-free residuals:

$$\boldsymbol{r}(k) = WC\left[V\Lambda_L^k P^T e(0) + \sum_{i=1}^{k} V\Lambda_L^{i-1}P^T\Delta e(k-i)\right] + Ww(k) \tag{48}$$

where $e(0) = \boldsymbol{x}(0) - \hat{\boldsymbol{x}}(0)$ is the initial error condition, $\Lambda_L = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, and $G_L = V\Lambda_L P^T$, where P and V are the modal matrices corresponding to G_L. By a way similar to Sobel and Banda's[21] method, taking the 2-norm of both sides of Eq. (48) we obtain

$$\boldsymbol{r}(k) \le \|P^T\|_2\cdot\|V\|_2\cdot\|WC\|_2\cdot\Delta(k) + \|W\|_2\|w(k)\|_2 \tag{49}$$

where

$$\Delta(k) = \left\|\Lambda_L^k\right\|_2\cdot\|e(0)\|_2 + \sum_{i=1}^{k}\left\|\Lambda_L^{i-1}\right\|_2\cdot\|\Delta e(k-i)\|_2$$

and $\| \; \|_2$ is defined as the induced 2-norm. Because $P^T V = I$, here $\|P^T\|_2\cdot\|V\|_2 \equiv \mathrm{K}(P)$ (Ref. 24) is known as the condition number of P. Equation (49) indicates that if the condition number of P is minimized, the 2-norm of the residual vector due uncertainties and initial condition mismatches can be reduced.

As for eigenstructure assignment, the objective in eigenvector selection here should be to make the eigenvectors as nearly mutually orthogonal as possible, so as to reduce $\mathrm{K}(P)$ (Ref. 24). We then have the multiobjective problem of optimizing the condition number by eigenstructure assignment while simultaneously satisfying the constraints on the allowable subspaces for multiple fault isolation in Eqs. (11) and (12) and the existence condition of the projection matrix W in Lemma 1, i.e., $N(C)\cap N(P_2^T) = \{0\}$. Based on the rank-one updating method addressed by Kautsky et al.,[25] we present a numerical algorithm to approximate the minimum condition number following the numerical results derived in preceding section. The resulting eigenstructure is made as orthonormal as possible, which approaches the minimum condition number $\mathrm{K}(P)$.

Algorithm 2: The bases N_{λ_i} and M_{λ_i} for $\boldsymbol{p}_i$ and ξ_i, $\forall\lambda_i \in \sigma(G - LC)$, for $i = 1, 2, \ldots, n$, are the given data.

1) Obtain N_{λ_i} as orthonormal bases $\bar{N}_{\lambda_i}$, i.e., $\bar{N}_{\lambda_i}^T\bar{N}_{\lambda_i} = I_{n_i}$ with rank $(N_{\lambda_i}) = n_i$. Define

$$P_{c_1,c_2,\ldots,c_n} = \left[\left\langle\bar{N}_{\lambda_1}\right\rangle_{c1} \quad \left\langle\bar{N}_{\lambda_2}\right\rangle_{c2} \quad \cdots \quad \left\langle\bar{N}_{\lambda_n}\right\rangle_{cn}\right] \tag{50}$$

where $\langle\bar{N}_{\lambda_i}\rangle_{c_i}$ denotes the c_ith column of the bases $\bar{N}_{\lambda_i}$. Determine $(\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_n)$ such that

$$\mathrm{K}\left(P_{\bar{c}_1,\bar{c}_2,\ldots,\bar{c}_n}\right) = \min_{c_i\in n_i}\mathrm{K}\left(P_{c_1,c_2,\ldots,c_n}\right) \tag{51}$$

The initial condition for the iteration is determined as

$$X = P_{\bar{c}_1,\bar{c}_2,\ldots,\bar{c}_n} = [\boldsymbol{\eta}_1 \quad \boldsymbol{\eta}_2 \quad \cdots \quad \boldsymbol{\eta}_n] \tag{52}$$

2) Update well-conditioned modal matrix P. Let

$$X^i = [\boldsymbol{\eta}_1 \quad \cdots \quad \boldsymbol{\eta}_{i-1} \quad \boldsymbol{\eta}_{i+1} \quad \cdots \quad \boldsymbol{\eta}_n]$$

and $\boldsymbol{\eta}_i^{\perp}$ be a vector orthogonal to X^i with $\|\boldsymbol{\eta}_i^{\perp}\|_2 = 1$. Compute

$$\hat{\boldsymbol{\eta}}_i = \left\{\left(\bar{N}_{\lambda_i}\right)^T\left(\bar{N}_{\lambda_i}\right)\right\}^{-1}\left(\bar{N}_{\lambda_i}\right)^T\boldsymbol{\eta}_i^{\perp} \tag{53}$$

Set $\hat{X}^i = [\boldsymbol{\eta}_1 \;\cdots\; \boldsymbol{\eta}_{i-1} \;\hat{\boldsymbol{\eta}}_i\; \boldsymbol{\eta}_{i+1} \;\cdots\; \boldsymbol{\eta}_n]$ and find $\bar{i}$ such that

$$\mathrm{K}(\hat{X}^{\bar{i}}) = \min_{i\in n}\mathrm{K}(\hat{X}^i) \tag{54}$$

To prevent the iteration from becoming unstable, check whether $\mathrm{K}(X) - \mathrm{K}(\hat{X}^{\bar{i}}) >$ tolerance or not: If so, reconstruct X by replacing the $\boldsymbol{\eta}_{\bar{i}}$ with $\hat{\boldsymbol{\eta}}_{\bar{i}}$; if $N(C)\cap N([\boldsymbol{\eta}_{p+1} \;\; \boldsymbol{\eta}_{p+2} \;\; \cdots \;\; \boldsymbol{\eta}_n]^T) = \{0\}$, register $P = X$. Repeat step 2. Otherwise, quit step 2.

3) The approximated solution for well-conditioned eigenvector assignment is P. Consequently, $[\xi_1 \;\; \xi_2 \;\; \cdots \;\; \xi_n]$ is given by

$$\boldsymbol{\alpha}_i = \left(N_{\lambda_i}^T N_{\lambda_i}\right)^{-1}N_{\lambda_i}^T\boldsymbol{p}_i, \qquad \boldsymbol{\xi}_i = M_{\lambda_i}\boldsymbol{\alpha}_i, \quad \text{for} \quad i = 1, 2, \ldots, n \tag{55}$$

After the column vectors $\boldsymbol{\alpha}_i$ have been determined, L can be obtained by following step 4 and step 5 of Algorithm 1, which yields a well-conditioned diagnostic observer. Thus, the combination of Algorithms 1 and 2 satisfactorily solve the problem of designing L and W such that $\boldsymbol{G}_{rf}(z)$ in Eq. (10) is diagonalized and the upper bound of $\|\boldsymbol{r}(k)\|_2$ due to unstructured uncertainties in Eq. (49) is reduced as much as possible without loss of fault detection sensitivity.

Remark 3: To prevent the condition $N(C)\cap R(P_2^T) = \{0\}$ from interrupting the iteration for updating well-conditioned eigenvectors, in the step 2 of Algorithm 2 a register is used to record the P that satisfies Lemma 1 without interrupting the procedure so that the iteration can continue running.

Remark 4: To design a residual generator in the state space, there are two major methods: observer-based methods and parity relation method.[26] Results show a systematic design and numerical solution to choose the observer gain L and projection matrix W so that mutual fault isolation and robustness to unstructured uncertainty can be achieved simultaneously. The parity relation methods, as studied by Lou et al.,[15] are other methods that can solve fault isolation problem by designing a structured residual set.[7,15] However, it is not very easy to design a fixed-directional residual vector using the parity relation method.[22] Also, the parity equation approaches adopted so far describe the solution of the FDI problem using a dead-beat observer algorithm.[4,13] The results imply that the parity relation method provides less design flexibility when compared with methods that are based on an observer. Another relation between observer-based methods and the generalized parity relation method using a frequency-domain approach has been investigated by many researchers.[4,6,7] Despite the internal structure, the diagnostic observer in Eqs. (3–5) can be written in the following input–output format:

$$\boldsymbol{r}(z) = H_u(z)\boldsymbol{u}(z) + H_y(z)\boldsymbol{y}(z) \tag{56}$$

$$= H_f(z)\boldsymbol{f}(z) \tag{57}$$

where

$$H_u(z) = -WC(zI - G + LC)^{-1}H$$

$$H_y(z) = W[I - C(zI - G + LC)^{-1}L]$$

$$H_u(z) = -WC[zI - (G - LC)]^{-1}K$$

In the preceding equations, Eq. (56) is the computational form and Eq. (57) is the internal form of the residual generator. In fact, by regarding the diagnostic observer, as in Eqs. (3–5), Eqs. (56) and (57) yield a group of generalized parity equations. Advantages of using eigenstructure assignment in the design of a diagnostic observer are that it provides a more systematic solution to the problem of robust multiple fault isolation than parity relation approaches can.

IV. Example: FDI System for a VTOL Aircraft

Consider the linearized dynamics of a VTOL aircraft in the vertical plane as proposed by Narendra and Tripathi.[27] The continuous-time state-space formulation is

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t)$$

where the states are the horizontal velocity (knot), vertical velocity (knot), pitch rate (degree/second), and pitch angle (degree), respectively; the actuator input u_1 is the collective pitch control and u_2 is the longitudinal pitch control. The system matrices are given with 1-ms sampling period as

$$G = \begin{bmatrix} 1 & 0 & 0 & -0.0005 \\ 0 & 0.9990 & 0 & -0.0040 \\ 0.0001 & 0.0004 & 0.9993 & 0.0014 \\ 0 & 0 & 0.0010 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.0004 & 0.0002 \\ 0.0035 & -0.0076 \\ -0.0055 & 0.0045 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

The present example considers the design of residual generator for multiple actuator fault isolation and, thus, $K = H$. The open-loop eigenvalues of the discrete-time nominal plant are located at $\{0.9979, 0.9998, 1.0003 \pm 0.0003i\}$. A stabilized controller using state feedback with pole placement is directly employed to stabilize the aircraft. These dynamics hold for typical loading and flight conditions at an air speed of 135 kn. To verify the robustness of the proposed FDI, the airspeed is assumed to vary randomly around the operating point. Following Algorithm 1, the design procedure is as follows.

1) Assign the eigenvalue set of the diagnostic observer as $\mathcal{L} = \{0.1, 0.2, 0.3, 0.4\}$.

2) Obtaining T_K, we have $\tilde{G} = T_K G T_K^{-1}$ and $\tilde{C} = C T_K^{-1}$.

3) and 4) Arbitrarily choose α_i for $i = 1, 2, \ldots, n$, such that $p_i = N_{\lambda_i}\alpha_i$ fulfills conditions a and b of step 3 in Algorithm 1 and $\xi_i = M_{\lambda_i}\alpha_i$. We obtain

$$P = \begin{bmatrix} 0.0370 & -0.6970 & -0.7790 & -0.8059 \\ 0.3675 & -0.1270 & -0.0886 & -0.0917 \\ 0.6201 & -0.1373 & -0.1194 & -0.1236 \\ -0.0330 & -0.2894 & 0.1231 & 0.1672 \end{bmatrix}$$

5) Compute

$$L = \begin{bmatrix} 1.5866 & 0.2993 & 0.3251 & -0.2174 \\ -25.0930 & -8.7954 & -10.6292 & 6.6778 \\ 14.9391 & 5.7745 & 7.2302 & -3.9788 \\ 2.0321 & -0.0467 & 0.0347 & 0.2779 \end{bmatrix}$$

6) Calculate

$$W = \Lambda_W (CV_1)^{+} = \begin{bmatrix} 20.4713 & -15.5016 & -122.0990 & -137.6617 \\ 11.0770 & -76.6640 & -15.3581 & -92.0297 \end{bmatrix}$$

The resulting diagnostic observer is in the form of Eqs. (3–5) and the transfer matrix from the faults to the residual vector is

$$\boldsymbol{G}_{rf}(z) = \begin{bmatrix} \dfrac{0.9}{(z-0.1)} & 0 \\ 0 & \dfrac{0.8}{(z-0.2)} \end{bmatrix} \tag{58}$$

Note that the condition number $\mathrm{K}(P)$ of the present design is about 817.5. To obtain a robust multiple fault isolation observer, Algorithm 2 is employed to determine a well-conditioned eigenvector set

$$P = \begin{bmatrix} 0.1864 & -0.1750 & 0.9706 & -0.1962 \\ -0.4969 & 0.8348 & 0.1105 & -0.0223 \\ -0.8476 & 0.5220 & 0.1488 & -0.0296 \\ -0.0004 & 0.0003 & -0.1534 & -0.9799 \end{bmatrix}$$

with $\mathrm{K}(P) = 4.1$. The corresponding L and W are given as

$$L = \begin{bmatrix} 0.7033 & -0.0034 & -0.0124 & -0.0153 \\ 0.0108 & 0.7448 & -0.0961 & -0.0058 \\ -0.0494 & 0.0907 & 0.9536 & -0.0007 \\ -0.0194 & -0.6052 & -0.6049 & 0.6031 \end{bmatrix}$$

$$W = \Lambda_W\left(V_1^{+}\right)(C^{-1}) = \begin{bmatrix} 55.9466 & -148.9992 & -254.2670 & -0.1272 \\ 34.7930 & -165.9431 & -103.7483 & -0.0519 \end{bmatrix}$$

Note that because $C \in R^{4\times 4}$ is a square and nonsingular matrix in this example, we choose $W = \Lambda_W(V_1^{+})(C^{-1})$ instead of $W = \Lambda_W(CV_1)^{+1}$ to further reduce the uncertainty effect due to the terms of $\|WC\|_2$ and $\|W\|_2$, as shown in Eq. (49). We emphasize that the transfer matrix from the faults to the residual vector is the same as that in Eq. (58), whereas the condition number of the modal matrix of $(G - LC)$ is reduced from 817.5 to 4.1. The aircraft was simulated with an initial state mismatch of $[0.05\ \ 0.05\ \ 0\ \ -0.05]$. The residual response of the observer with $\mathrm{K}(P) = 817.5$ is shown in Fig. 2, where a bias fault of collective pitch control occurs after sampling index $k = 60$ with magnitude two and another negative bias fault of magnitude three acts on the longitudinal pitch control after $k = 120$. Figure 3 shows the residual outputs of the well-conditioned design with $\mathrm{K}(P) = 4.1$ under the same operating conditions. In the case of Fig. 3, despite the uncertainty, a threshold can easily be placed on the residual signal to declare the occurrence of faults, whereas

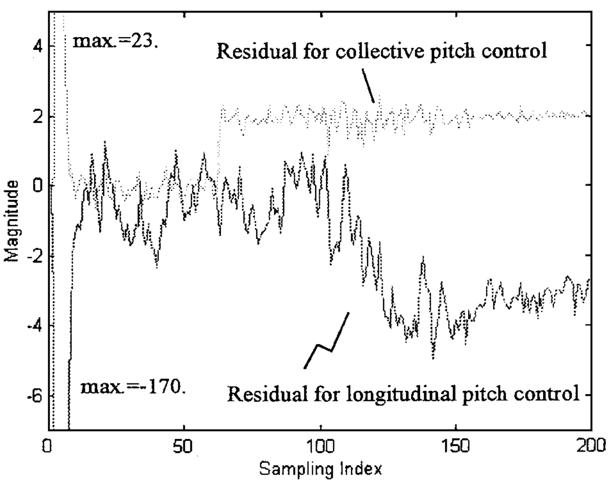

Fig. 2 Residual responses for multiple actuator faults of an ill-conditioned FDI design [K(*P*) = 817.5].

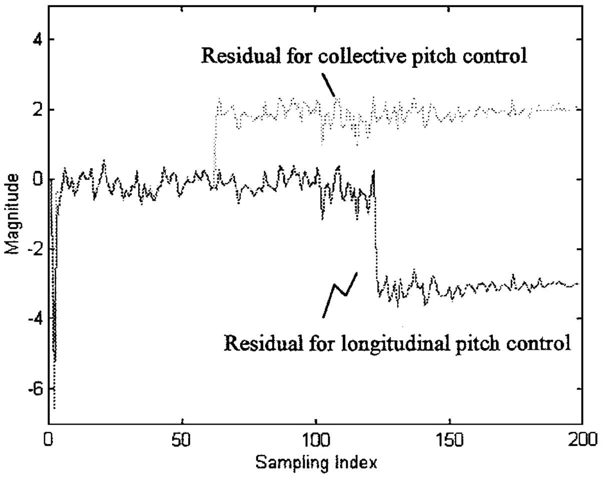

Fig. 3 Residual responses for multiple actuator faults of a well-conditioned FDI design [K(*P*) = 4.1].

in the case of Fig. 2, a tradeoff between missing alarm and false alarm must be made when the threshold is placed. In other words, the uncertainty effect to fault ratio in the well-conditioned design is comparatively smaller than that in the ill-conditioned design; the former FDI system is much more suitable and reliable to detect soft failures than the latter. At the same time, it is seen that the well-conditioned FDI is also robust to the initial condition mismatch. The results show that the fault detection sensitivity of the residuals is maintained without any degradation when the uncertainty effect is suppressed.

V. Design of a Reduced-Order Fault Detection and Isolation Observer

Inasmuch as the purpose of a diagnostic observer is to achieve fault diagnosis instead of state estimation, unnecessary computation or fault invariant states can be neglected. Thus, we will now consider the design of a reduced-order robust diagnostic observer. Suppose that (C, G) has $\bar{o}$ unobservable modes, and let $\hat{\boldsymbol{x}}(k) = V\hat{z}(k)$; Eqs. (3–5) can then be transformed as follows:

$$\begin{aligned}\hat{z}(k+1) &= P^T G_L V\hat{z}(k) + P^T H\boldsymbol{u}(k) + P^T L\boldsymbol{y}(k)\\ &= \Lambda_L\hat{z}(k) + \hat{H}\boldsymbol{u}(k) + \hat{L}\boldsymbol{y}(k) \qquad (59)\end{aligned}$$

$$\hat{\boldsymbol{y}}(k) = CV\hat{z}(k) \qquad (60)$$

$$\begin{aligned}\boldsymbol{r}(k) &= -WCV\hat{z}(k) + W\boldsymbol{y}(k)\\ &= -WC[V_1 \quad V_2 \quad V_3 \quad V_4]\hat{z}(k) + W\boldsymbol{y}(k) \qquad (61)\end{aligned}$$

where $V_1 \in R^{n\times p}$, $V_2 \in R^{n\times(n-r-\bar{o})}$, $V_3 \in R^{n\times(r-p)}$, and $V_4 \in R^{n\times\bar{o}}$. Because $C \in R^{r\times n}$ has full row rank, $\exists W \in R^{p\times r}$ satisfies $W[CV_1 \;\; CV_2] = [\Lambda_W \;\; 0]$. Together with the unobservable modes, $CV_4 = 0$, Eq. (61) becomes

$$\boldsymbol{r}(k) = -[\Lambda_W \quad WCV_2 \quad 0 \quad 0]\hat{z}(k) + W\boldsymbol{y}(k) \qquad (62)$$

Because Λ_L is diagonal, the reduced-order observer, thus, can be constructed by retracting the zero contribution states in Eq. (62) as

$$\begin{bmatrix}\hat{z}_1(k+1)\\ \vdots\\ \hat{z}_{n'}(k+1)\end{bmatrix} = \begin{bmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_{n'}\end{bmatrix}\begin{bmatrix}\hat{z}_1(k)\\ \vdots\\ \hat{z}_{n'}(k)\end{bmatrix} + \begin{bmatrix}\hat{H}_1^T\\ \vdots\\ \hat{H}_{n'}^T\end{bmatrix}\boldsymbol{u}(k) + \begin{bmatrix}\hat{L}_1^T\\ \vdots\\ \hat{L}_{n'}^T\end{bmatrix}\boldsymbol{y}(k) \qquad (63)$$

$$\boldsymbol{r}(k) = [\Lambda_W \quad WCV_2]\begin{bmatrix}\hat{z}_1(k)\\ \vdots\\ \hat{z}_{n'}(k)\end{bmatrix} + W\boldsymbol{y}(k)$$

where $n' = (n-\bar{o}) - (r-p)$, $\hat{z}_i$ is the ith state of $\hat{z}$ and $\hat{H}_i^T$, and $\hat{L}_i^T$ is the ith row of $\hat{H}$ and $\hat{L}$, respectively. A related procedure for reducing the order of a diagnostic observer was presented by Chang and Hsu.[26] For the example of the VTOL aircraft here, $n = 4$, $p = 2$, $r = 4$, and $\bar{o} = 0$, and W can be obtained from $W = [\Lambda_W \;\; 0]([V_1 \;\; V_3]^+)C^{-1}$. Consequently, the reduced-order robust observer is

$$\begin{aligned}\begin{bmatrix}\hat{z}_1(k+1)\\ \hat{z}_2(k+1)\end{bmatrix} &= \begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}\begin{bmatrix}\hat{z}_1(k)\\ \hat{z}_2(k)\end{bmatrix}\\ &+ \begin{bmatrix}-0.0030 & 0\\ 0 & -0.0040\end{bmatrix}\boldsymbol{u}(k)\\ &+ \begin{bmatrix}0.1676 & -0.4473 & -0.7625 & 0.0003\\ -0.1399 & 0.6695 & 0.4196 & -0.0023\end{bmatrix}\boldsymbol{y}(k)\end{aligned}$$

$$\begin{aligned}\boldsymbol{r}(k) &= \begin{bmatrix}300.1419 & 0\\ 0 & -198.8451\end{bmatrix}\begin{bmatrix}\hat{z}_1(k)\\ \hat{z}_2(k)\end{bmatrix}\\ &+ \begin{bmatrix}300.1419 & 0\\ 0 & -198.8451\end{bmatrix}\begin{bmatrix}\boldsymbol{y}_1(k)\\ \boldsymbol{y}_2(k)\end{bmatrix}\end{aligned}$$

where $\boldsymbol{y}_i(k)$ denotes the ith measurement of the output vector. The transfer matrix from fault to residual is diagonal, the same as in Eq. (58). The computational load of the full-order observer in this case is 56 multiplications and 40 additions, whereas the reduced-order observer design needs only 16 multiplications and 12 additions to provide identical diagnostic performance. The computational efficiency is significantly improved.

Remark 5: If $\{CV_3\} \cap R(CV_2) \neq \{0\}$, some columns of CV_2 will consist of all zeros. Then, more than $(r-p) + \bar{o}$ states can be retracted.

Remark 6: Suppose that there exists an undetectable mode λ_j of the pair (G, C). Theoretically, $C\boldsymbol{v}_j = 0$, which entails that the contribution of the jth mode λ_j from faults to residual vector

$$\frac{C\boldsymbol{v}_j\boldsymbol{p}_j^T K}{(z-\lambda_j)} = 0 \qquad (64)$$

Unfortunately, because of the limited accuracy of computers or microprocessors, $\boldsymbol{r}(k)$ will approach infinity as k approached infinity even without failure occurrence. To avoid the run-time error problem arising from the undetectability, the undetectable modes must be retracted using this order reduction procedure.

VI. Conclusion

Most approaches based on eigenstructure assignment for robust FDI are designed by assigning the eigenvectors orthogonal to the structured uncertainties to achieve disturbance decoupling. However, in practice, one critical limitation of these approaches to FDI arises as the uncertainties acting on the diagnostic observer are unstructured. We formulate the problem of multiple fault isolation in eigenstructure assignment and parameterize the allowable eigenspaces of the diagnostic observer as provided by Algorithm 1. By using this formulation, the robustness of the observer with respect to unstructured uncertainties is related to the condition number of the observer modal matrix, which yields an approximation procedure for the synthesis of a well-conditioned observer design as in Algorithm 2. By using these algorithms, the proposed diagnostic observer achieves both multiple faults isolation and robustness to modeling errors simultaneously. It should be emphasized that the robustness aspects of the present observer still guarantee the fault detection sensitivity without any degradation. Also, by eliminating those states of no use for FDI, a direct procedure for reducing the order of the diagnostic observer also has been developed to improve the computational efficiency and implementation feasibility. Desirable robust performance results of multiple faults isolation under unstructured uncertainties have been demonstrated using a simulation of a VTOL aircraft. Simulation results indicate that the diagnostic performance of an arbitrary observer design is significantly improved by a well-conditioned observer design.

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